Section 8.2: Systems of Linear Equations – Matrices

Steps to solve a system of 2 equations and 2 unknowns using matrices and Gauss Jordan Reduction.

Step 1) Create the matrix implied by the system of equations.

For example
$5x - 3y = 9$
$2x + 7y = 20$

Will become the matrix:

$$\begin{bmatrix} 5 & -3 & 9 \\ 2 & 7 & 20 \end{bmatrix}$$

Step 2: Use elimination method to eliminate the x's.

$-2(5 \quad -3 \quad 9)$
$5(2 \quad 7 \quad 20)$

$$= \begin{matrix} -10 & 6 & -18 \\ 10 & 35 & 100 \end{matrix}$$

$$= \begin{matrix} 0 & 41 & 82 \end{matrix}$$

Step 3: divide away the common factor

$$= \frac{0}{41} \quad \frac{41}{41} \quad \frac{82}{41}$$

$$= \begin{matrix} 0 & 1 & 2 \end{matrix}$$

Step 4: Make the answer to step 2 the bottom row of the matrix

$$\begin{bmatrix} 5 & -3 & 9 \\ 0 & 1 & 2 \end{bmatrix}$$

Step 5: Use the elimination method to eliminate the y's.

$5 \quad -3 \quad 9$
$3(0 \quad 1 \quad 2)$

=

$5 \quad -3 \quad 9$
$0 \quad 3 \quad 6$

=

$5 \quad 0 \quad 15$

Step 6: divide away the common factor:

$=\frac{5}{5} \quad \frac{0}{5} \quad \frac{15}{5}$

$= 1 \quad 0 \quad 3$

Step 7: Make the answer to step 6 the new top row for the matrix created in step 4

$\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \end{bmatrix}$

Step 8: Create the system of equations from the matrix created in step 6.

$1x + 0y = 3$

$0x + 1y = 2$

Step 9: Simplify the equations and write your answer.

$x = 3$
$y = 2$

Answer $(3,2)$

Step 10: Check
$5(3) - 3(2) = 9$ simplifies to 9 = 9 ✓
$2(3) + 7(2) = 20$ simplifies to 20 = 20 ✓

Solve the system of equations using matrices and row operations.

1) $\begin{aligned} 6x + 2y &= 10 \\ 2x - y &= 5 \end{aligned}$

2) $\begin{aligned} 8x - 3y &= -2 \\ 2x + y &= -4 \end{aligned}$

3) $\begin{aligned} 4x - 3y &= -2 \\ x - 5y &= -9 \end{aligned}$

4)

$\begin{aligned} 5x + 2y &= 16 \\ x - 7y &= -19 \end{aligned}$

5) $\begin{aligned} 5x + y &= -7 \\ 3x - 2y &= -12 \end{aligned}$

6) $\begin{aligned} 4x + y &= 11 \\ 3x - 5y &= 37 \end{aligned}$

7) $\begin{aligned} 3x + 2y &= 11 \\ 2x - y &= 5 \end{aligned}$

8) $\begin{aligned} 4x - 3y &= 10 \\ 2x + y &= 10 \end{aligned}$

9) $\begin{aligned} 4x-2y&=7\\ 2x-5y&=-3 \end{aligned}$

10) $\begin{aligned} 6x+2y&=26\\ 2x-7y&=24 \end{aligned}$

11) $\begin{aligned} 5x+2y&=22\\ 3x-5y&=7 \end{aligned}$

12)

$\begin{aligned} 4x+2y&=8\\ 3x-5y&=-7 \end{aligned}$

13)

$\begin{aligned} -x+y+2z&=1\\ 2x+3y+z&=-2\\ 5x+4y+2z&=4 \end{aligned}$

14)

$\begin{aligned} 3x-2y+z&=15\\ -x+y+2z&=-10\\ x-y-4z&=14 \end{aligned}$

15)

$\begin{aligned} -5x-y+3z&=-14\\ -2x+2y-6z&=16\\ x+7y+2z&=-5 \end{aligned}$

16)

$\begin{aligned} 4x+4y+4z&=12\\ 4x-2y-8z&=-12\\ 5x+3y+8z&=21 \end{aligned}$

17)

$$-x + 2y - z = -17$$
$$2x - y + z = 21$$
$$3x + 2y + z = 19$$

18)

$$x + y + 2z = 6$$
$$2x + 3y + z = 11$$
$$5x + 4y + 2z = 19$$

19)

$$4x + y + z = 9$$
$$3x - 2y + z = 4$$
$$5x - 4y + z = 6$$

20)

$$x - y + z = 2$$
$$2x + y + z = 5$$
$$7x + 4y - z = 9$$